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Non probabilistic reliability analysis model of structures based on rough set theory

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Abstract. There are various uncertain variables in engineering structure analysis; rough set theory can make the uncertainty of simulation structure easy, and describe the uncertain parameter as variable of rough interval. According to structural non-probability reliability analysis, it comes up with a type of new computing method of structural reliability index through taking advantage of interval's reliability analysis theory. This method fully considers the influence of uncertainty of variable on structure, improves the accuracy of structural reliability calculation, and provides analysis of examples.

Key words. Structural reliability index, Rough set theory, Rough variable, Non-probabilistic reliability.

1. Introduction

According to the definition of unified design standard [1] for structural reliability index, structural reliability index is that its ability to accomplish intended function within fixed time and under the specified condition and the relevant probability is reliability. The specific time means the design working life, namely, structures or components can accord with the cost time of intended purpose without heavy repair. The specified conditions are "three-normality", namely, normal design, normal construction and normal use for intended function, which also can be known as security, applicability and durability.

The traditional computing method which regards probability theory and mathematical statistics as foundation of mathematics can be called as probability reliabil-

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ity calculation method [2], and also can be known as common reliability calculation method. The common structural reliability calculation includes first-order second-moment method, high-order high-moment method, optimization method, response surface methodology, Monte Carlo method and stochastic finite element methods.

There are many uncertain factors in various engineering structures. For example, structural physical property, geometrical parameter, and its bearing load (such as wind load, sea load and seismic load and so on). Due to the limitation of conditions, people can not determine the numerical value in advance, and the uncertain factors cause increasingly serious influence. The traditional uncertain information processing method includes fuzzy set theory, evidence theory and probability and statistics theory and so on.

Document researches [3] indicate that probability reliability calculation is very sensitive to probabilistic model parameter, and the minor error of probabilistic data can lead to the larger error of calculation of structural reliability, so the probability reliability theory faces great challenge. This text takes advantage of rough set theory to calculate structural reliability index, uses the uncertainty of theoretical simulation of rough set theory to describe the uncertain parameter of structure as rough variable, and adopts the maximum degree of uncertainty allowed by structure to measure reliability, thereby, obtaining structural reliability index which is more accurate on representations of variable and result.

2. Basic theory of rough set

Rough set theory, a theory to deal with inaccurate, uncertain and incomplete data, can effectively analyze various inaccurate, inconsistent and incomplete information and it also a type of powerful tool about data inference, which is first put forward by Poland scientist Pawlak in 1982. It mainly has the following advantages: it only relies on the original data, and does not need any external information and prior information, such as probability distribution in statistics and degree of membership in fuzzy set theory and so on, so its description or processing for the uncertainty of problems is more subjective; although its mathematical method to process inaccurate and fuzzy problems is to calculate the approximate value, such as fuzzy sets and evidence theory, etc., its difference is that rough set obtains approximate value through mathematical formula, but the fuzzy set theory only can rely on statistical approach to get the approximate results; it is not only suitable for the analysis of quality attribute but suitable for the analysis of quantity attribute; it can simplify the redundancy property and the reduction algorithm is simpler, namely, the decision rule set exported by rough set model provides the minimum knowledge representation; it does not correct inconsistencies and divides the generated inconsistent rules into certain rules and possible rules; the results are easy to understand; it can find the abnormality in the data and eliminates the noise interference in the process of knowledge discovery; it is good for parallel execution and improves detection efficiency; compared with the fuzzy set method or neural network method, the decision-making rules and reasoning processes obtained by the rough set method are more easily verified and explained.

Definition 1 [5] will provide a knowledge representation system $S = (U, A, V, f), P \subseteq A, X \subseteq U, x \in U$, the set X's lower approximation, upper approximation, minus zone and approximate precision related to I respectively are:

$$\underline{apr}_P(X) = \bigcup \{x \in U : I(x) \subseteq X\}\,. \tag{1}$$

$$\overline{apr}_P(X) = \bigcup \{x \in U : I(x) \cap X \neq \Phi\}\,. \tag{2}$$

$$neg_P(X) = \bigcup \{x \in U : I(x) \cap X = \Phi\}\,. \tag{3}$$

$$bnd_P(X) = \overline{apr}_P(X) - \underline{apr}_P(X)\,. \tag{4}$$

$$\alpha_P(X) = \frac{|\underline{apr}_P(X)|}{|\overline{apr}_P(X)|}\,. \tag{5}$$

We can know the following formulas about the fuzzy set FX through the definition of rough-fuzzy set [6]

$$\underline{apr}_P(FX) = \bigcup \{x \in U : \mu_{FX}(x) = 1\}\,. \tag{6}$$

$$\overline{apr}_P(FX) = \bigcup \{x \in U : \mu_{FX}(x) > 0\}\,. \tag{7}$$

And then get the following formulas through the definition of fuzzy set [7]

$$\ker(FX) = \underline{apr}_P(FX)\,. \tag{8}$$

$$\sup p(FX) = \overline{apr}_P(FX)\,. \tag{9}$$

Finally get the following formulas about random variable SX through the definition of rough membership function and variable precision rough sets model

$$\underline{apr}_P(SX) = \bigcup \{x \in U : \mu_{SX}(x) \geq \beta\}\,. \tag{10}$$

$$\overline{apr}_P(SX) = \bigcup \{x \in U : \mu_{SX}(x) > 1 - \beta\}\,. \tag{11}$$

Thus, we do not have to consider the subordinate function of fuzzy variable and probability of random variable when make the structural reliability analysis, and can express the uncertain variables with different types as unified rough set variable, which makes the calculation easier.

3. Structural reliability analysis based on rough set

Suppose $X = (X_1, X_2, \cdots, X_n)^T (X_i = [a_i, b_i],\ i \in n)$ is n rough variables to affect structural, and X can be the structural physical dimension, material's physical and mechanics parameters and the function affected by structure, etc. We call the random function as structural function (or failure function).

$$G = g(X) = g(X_1, X_2, \cdots, X_n)\,. \tag{12}$$

G is the rough variable through calculation, and the mean value and deviation respectively are G^c and G^r, suppose

$$Z = \frac{G^c}{G^r}\,. \tag{13}$$

According to internal's reliability analysis model [9], we can know that when $Z > 1$ represents the reliable state of structure and $Z \leq -1$ represents its failure state, $-1 < Z \leq 1$ means that it is in reliable state or failure state, namely, uncertain state, in the strict sense, it is in the failure state.

3.1. Function of double rough variables

Function

$$G = X_1 - X_2 = 0\,. \tag{14}$$

X_1 and X_2 respectively are the structural rough variable of strength and stress. According the analysis in the preceding part of the text, suppose

$$X_i^c = \underline{X_i}\,. \tag{15}$$

$$X_i^r = \overline{X_i}\,. \tag{16}$$

And get

$$\begin{cases} Z = \dfrac{\underline{X_1} - \underline{X_2}}{\overline{X_1} + \overline{X_2}} & \underline{X_1} > \underline{X_2} \\ 0 & otherwise \end{cases} \tag{17}$$

3.2. Function of multiple rough variables

Function

$$G = \sum_{i=1}^{m} a_i X_{1i} - \sum_{i=m+1}^{n} b_i X_{2i} = 0\,. \tag{18}$$

X_{1i} and X_{2i} respectively are rough variables which are not related to the structure, So we can get

$$\begin{cases} Z = \dfrac{\sum\limits_{i=1}^{m} a_i \underline{X_{1i}} - \sum\limits_{i=m+1}^{n} b_i \underline{X_{2i}}}{\sum\limits_{i=1}^{m} |a_i| \overline{X_{1i}} + \sum\limits_{i=m+1}^{n} |b_i| \overline{X_{2i}}} & \sum\limits_{i=1}^{m} a_i \underline{X_{1i}} - \sum\limits_{i=m+1}^{n} b_i \underline{X_{2i}} > 0 \\ 0 & otherwise \end{cases} \tag{19}$$

3.3. Nonlinear performance function of rough variable

For the general nonlinear performance function, suppose

$$\begin{cases} Z = \min(||\delta||_\infty) = \min(\max\{|X_1'|, |X_2'|, \cdots, |X_3'|\}), \\ G = g(X) = g(X_1, X_2, \cdots, X_n) = 0, \end{cases} \tag{20}$$

$X_i' = \overline{X_i}'$ is the rough variable of standardization. According to interval's reliability analysis model we can know that the larger Z is, the further the failure domain of structural distance gets, and the structure is in the reliable state.

4. Analysis of examples

Example 1 linear function $G = R_1 + R_2 - 0.1S$
Decision information table 1 of rough variable is as follows:

Table 1. Attribute decision information of State Function

U	R_1	R_2	S
a	0.5	0.015	0.127
b	0.265	0.017	0.136
c	0.375	0.0162	0.180
⋮	⋮	⋮	⋮

$\underline{R_1} \in [0.5]$, $\overline{R_1} \in [0.275, 0.505]$, $\underline{R_2} \in [0.016]$, $\overline{R_2} \in [0.015, 0.018]$, $\underline{S} \in [0.1]$, $\overline{S} \in [0.126, 0.182]$ are unrelated rough variables.

$$Z = \frac{0.5 + 0.016 - 0.1 \times 0.1}{[0.275, 0.505] + [0.015, 0.018] + 0.1 \times [0.126, 0.182]} = [1, 1.433]$$

Example 2 considers certain nonlinear limit state function $G = 567RS - 0.5H^2$. In, $\underline{R} \in [0.627]$, $\overline{R} \in [0.582, 0.672]$, $\underline{S} \in [2.18]$, $\overline{S} \in [2.071, 2.289]$, $\underline{H} \in [32.8]$,

$\overline{H} \in [31.16, 34.44]$ are unrelated rough variables.

$$\begin{cases} Z = \min(||\delta||_\infty) = \min(\max\{|R'|, |S'|, \cdots, |H'|\})\,, \\ 567RS - 0.5H^2 = 0\,. \end{cases}$$

$$\overline{R} = 0.627 + 0.09R'\,.$$

$$\overline{S} = 2.18 + 0.109S'\,.$$

$$\overline{H} = 32.8 + 1.64H'\,.$$

Substitute into the conditional equality, we can get

$$567(0.627 + 0.09R')(2.18 + 0.109S') - 0.5(32.8 + 1.64H')(32.8 + 1.64H') = 0\,.$$

The result is

$$Z = 1.56\,.$$

5. Conclusion

There are many uncertainties in a complex structure; how to use a method to analyze structural reliability accurately is the key in engineering calculation. It uses rough set theory to analyze fuzzy and random uncertainty factors, and adopts the relationship between upper and lower approximation to calculate the value interval of reliability index and analyze the reliability index. The characteristics of this type of analytical method is that it doesn't have to distinguish whether uncertainty parameters belong to fuzzy or random range, and doesn't have to determine the membership function about fuzzy variables or distribution function of random variable, just get reasonable values of variable through the objective rough analysis; compared to the non-probability reliability analysis, it does not have to make the interval's standardized calculation for uncertain parameters and can carry out the reliability analysis by using the objective rough analysis results.

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